UNIVERSIDADE FEDERAL DO PARANÁ
SETOR DE CIÊNCIAS EXATAS
DEPTO. DE ESTATÍSTICA

4ª LISTA DE EXERCICIOS CE330 – FUNDAM. MAT. PARA PROBABILIDADE

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SEQUÊNCIAS.

1. Decida se cada uma das sequências abaixo é convergente ou divergente, calculando o limite no caso de ser convergente.

(1) $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

(2) $1, \frac{1}{2}, 1, \frac{1}{4}, 1, \frac{1}{8}, 1, \frac{1}{16}, \ldots$

(3) $\frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{3}{4}, \frac{1}{8}, -\frac{7}{8}, \ldots$

(4) $a_n = \left(4 + \frac{1}{n}\right)^{\frac{1}{2}}$

(5) $a_n = \frac{\sqrt{n}+1}{n-1},\ n \geq 2$

(6) $a_n = \frac{n^3+3n+1}{4n^3+2}$

(7) $a_n = \sqrt{n+1} - \sqrt{n}$

(8) $a_n = \frac{n+(-1)^n}{n-(-1)^n}$

(9) $a_n = \frac{2n}{n+1} - \frac{n+1}{2n}$

(10) $a_n = n(\sqrt{n^2+1} - n)$

(11) $a_n = \frac{\operatorname{sen} n}{n}$

(12) $a_n = \operatorname{sen} n$

(13) $a_n = \frac{2n+\operatorname{sen} n}{5n+1}$

(14) $a_n = \frac{(n+3)!-n!}{(n+4)!}$

(15) $a_n = \sqrt[n]{n^2+n}$

(16) $a_n = \frac{n\operatorname{sen}(n!)}{n^2+1}$

(17) $a_n = \frac{3^n}{2^n+10^n}$

(18) $a_n = \left(\frac{n+2}{n+1}\right)^n$

(19) $a_n = \frac{(n+1)^n}{n^{n+1}}$

(20) $a_n = na^n,\ a \in \mathbb{R}$

(21) $a_n = \frac{n!}{n^n}$

(22) $a_n = n - n^2 \operatorname{sen} \frac{1}{n}$

(23) $a_n = \sqrt[n]{a^n + b^n},\ 0 < a < b$

(24) $a_n = (-1)^n + \frac{(-1)^n}{n}$

(25) $a_n = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\ldots\left(1 - \frac{1}{n}\right)$

(26) $a_n = \frac{\sqrt{n}+\operatorname{sen}(2n!-7)}{n+3\sqrt{n}}$

(27) $a_n = \frac{1}{n} \cdot \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots(2n)}$

(28) $a_n = \sqrt[n]{n}$

(29) $a_n = \frac{n^\alpha}{e^n},\ \alpha \in \mathbb{R}$

(30) $a_n = \frac{\ln n}{n^\alpha},\ \alpha > 0$

(31) $a_n = \sqrt[n]{n!}$

(32) $a_n = \sqrt[n]{a},\ a > 0$

(33) $a_n = \left(\frac{n-1}{n}\right)^n$

(34) $a_n = \left(\frac{n+1}{n}\right)^{n^2}$

(35) $a_n = \left(\frac{n+1}{n}\right)^{\sqrt{n}}$

(36) $a_n = \left(\frac{3n+5}{5n+11}\right)^n$

(37) $a_n = \left(\frac{3n+5}{5n+1}\right)^n \left(\frac{5}{3}\right)^n$

(38) $a_n = \left(1 + \frac{1}{n^2}\right)^n$

(39) $a_n = \operatorname{sen}\left(\frac{n\pi}{2}\right)$

(40) $a_n = \frac{n}{\sqrt[n]{n!}}$

(41) $a_n = \frac{1}{n}\sqrt[n]{(n+1)(n+2)\cdot\ldots\cdot(2n)}$

(42) $a_n = \sqrt[n]{\frac{(2n)!}{n!^2}}$

(43) $a_n = \frac{n^2-1}{n^5+(-1)^n n^2}$

(44) $a_n = \sqrt[n]{n^4 + 2012n^3 - 5}$

(45) $a_n = \left(1 + \frac{1}{n}\right)^{1/n}$

(46) $a_n = \frac{n!^2}{n^{2n}}$

(47) $a_n = \frac{5^n}{2^n+3^n+4^n}$

(48) $a_n = \frac{n+\sqrt{2n+3}}{\sqrt[4]{n}+\sqrt[7]{17n-8}}$

(49) $a_n = \frac{3n^3-n^2+11n}{n^4-2n^3}$

(50) $a_n = \left(\frac{5n+7}{3n+8}\right)^{2n-4}$

(51) $a_n = \frac{(2n)!}{(n!)^2}$

SÉRIES.

2. Decida se cada uma das séries abaixo é convergente e calcule sua soma quando possível:

(1) $\sum_{n=0}^{\infty}\left(\frac{1}{10^n}+2^n\right)$ (2) $\sum_{k=0}^{\infty}(-1)^k\, t^{\frac{k}{2}},\ 0<t<1$ (3) $\sum_{n=0}^{\infty} u^n(1+u^n),\ |u|<1$

(4) $\sum_{n=0}^{\infty} x^n\cos\left(\frac{n\pi}{2}\right),\ |x|<1$ (5) $\sum_{n=0}^{\infty}\operatorname{sen}^{2n}x,\ |x|<\frac{\pi}{2}$ (6) $\sum_{n=1}^{\infty}\frac{1}{e^{n/2}}$

(7) $\sum_{n=1}^{\infty}\cos\left(\frac{n\pi}{2}\right)$ (8) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$ (9) $\sum_{n=1}^{\infty}\frac{n}{\operatorname{sen} n}$

(10) $\sum_{n=1}^{\infty}\frac{n+2}{\sqrt{n+2012}}$ (11) $\sum_{n=1}^{\infty}\frac{2+\cos n}{n}$ (12) $\sum_{n=1}^{\infty}\operatorname{tg}\left(\frac{1}{n}\right)$

3. Verifique se cada uma das séries abaixo é convergente ou divergente, justificando sua resposta:

(1) $\sum_{n=3}^{\infty}\frac{1}{\sqrt{n^2-4}}$ (2) $\sum_{n=2}^{\infty}\frac{\arctan n}{n^2}$ (3) $\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}$ (4) $\sum_{n=1}^{\infty}\frac{2^n}{(n!)^{\lambda}},\ \lambda>0$

(5) $\sum_{n=1}^{\infty}\frac{(2n)!}{n!^2}$ (6) $\sum_{n=2}^{\infty}\frac{\ln n}{n}$ (7) $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n+2}}{\sqrt[4]{n^3+3}\,\sqrt[5]{n^3+5}}$ (8) $\sum_{n=2}^{\infty}\frac{1}{n^{\ln n}}$

(9) $\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right)$ (10) $\sum_{n=2}^{\infty}\frac{1}{(\ln\ n)^n}$ (11) $\sum_{n=2}^{\infty}\frac{\ln\ n}{n^2}$ (12) $\sum_{n=2}^{\infty}\frac{\ln n}{n^p},\ p>0$

(13) $\sum_{n=2}^{\infty}\ln\left(1+\frac{1}{n^p}\right),\ p>0$ (14) $\sum_{n=2}^{\infty}\sqrt{n}\ln\left(\frac{n+1}{n}\right)$ (15) $\sum_{n=1}^{\infty}\frac{n!3^n}{n^n}$ (16) $\sum_{n=1}^{\infty}\frac{n!e^n}{n^n}$

(17) $\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^n}$ (18) $\sum_{n=1}^{\infty}3^n\left(\frac{n}{n+1}\right)^{n^2}$ (19) $\sum_{n=1}^{\infty}\frac{n^3}{(\ln 2)^n}$ (20) $\sum_{n=1}^{\infty}\frac{1}{(\arctan n)^n}$

(21) $\sum_{n=0}^{\infty}\frac{n+2}{(n+1)^3}$ (22) $\sum_{n=1}^{\infty}\left(\sqrt[n]{2}-1\right)$ (23) $\sum_{n=1}^{\infty}\operatorname{sen}\left(\frac{1}{n}\right)$ (24) $\sum_{n=0}^{\infty}\frac{1+2^n}{1+3^n}$

(25) $\sum_{n=0}^{\infty}\frac{n}{(1+n^2)^p},\ p>0$ (26) $\sum_{n=1}^{\infty}\frac{1}{n+\sqrt[17]{n}}$ (27) $\sum_{n=0}^{\infty}\left(\frac{2n+1}{3n+4}\right)^n$ (28) $\sum_{n=0}^{\infty}\frac{\operatorname{sen}4n}{4^n}$

(29) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p},\ p>0$ (30) $\sum_{n=1}^{\infty}\ln(\cos(1/n))$ (31) $\sum_{n=0}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$ (32) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

(33) $\sum_{n=1}^{\infty}\frac{1}{(n\ln n)^p},\ p>0$ (34) $\sum_{n=1}^{\infty}(\sqrt{1+n^2}-n)$ (35) $\sum_{n=1}^{\infty}\frac{\ln n}{n^p e^n},\ p>0$ (36) $\sum_{n=0}^{\infty}e^{-n}n!$

(37) $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^p},\ p>0$ (38) $\sum_{n=1}^{\infty}\operatorname{sen}\left(\frac{1}{n\sqrt[4]{n^3+6}}\right)$ (39) $\sum_{n=1}^{\infty}\frac{\sqrt[8]{n^7+3n^3-2}}{\sqrt[6]{n^9+7n^2}}$ (40) $\sum_{n=0}^{\infty}\frac{n^2 2^n}{n!}$

(41) $\sum_{n=1}^{\infty}\frac{n^p}{e^{-an}},\ a,p>0$ (42) $\sum_{n=1}^{\infty}a^n n^p,\ a,p>0$ (43) $\sum_{n=1}^{\infty}\frac{(2n)^n}{n^{2n}}$ (44) $\sum_{n=2}^{\infty}\frac{1}{(\ln n)^{\ln n}}$

(45) $\sum_{n=1}^{\infty}\frac{1}{n^{1+1/n}}$ (46) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{2012}e^{-n/3}$ (47) $\sum_{n=1}^{\infty}\frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$ (48) $\sum_{n=1}^{\infty}\frac{n^n}{(n+1)^{n+1}}$

✩ Respostas

(1)

- (2), (3), (12), (24), (39) são divergentes;
- (5), (7), (11), (14), (16), (17), (19), (21), (22), (25), (26), (27), (29), (30), (36), (43), (46), (49) convergem para zero;
- (1), (8), (15), (28), (32), (35), (38), (44), (45) convergem para 1;
- (31), (34), (47), (48), (50), (51) divergem para $+\infty$;
- (4) converge para 2; (6) converge para $1/4$; (9) converge para $3/2$; (10) converge para $1/2$; (13) converge para $2/5$; (18) converge para e; (20) diverge se $a < 0$, diverge para $+\infty$ se $a \geq 1$ e converge para zero se $0 \leq a < 1$; (23) converge para b; (33) converge para $1/e$; (37) converge para $e^{22/15}$; (40) converge para e; (41) converge para $4/e$; (42) converge para 4.

(2)

(1) diverge; (2) converge para $\frac{1}{\sqrt{1+t}}$; (3) converge para $\frac{1}{1-u} + \frac{1}{1-u^2}$; (4) converge para $\frac{1}{1+x^2}$; (5) converge para $\sec^2 x$; (6) converge para $\frac{1}{\sqrt{e}-1}$; as demais são todas divergentes.

(3) (1), (5), (6), (14), (15), (16), (18), (19), (22), (23), (26), (34), (36),(39), (45), (47) são divergentes; (12), (13), (25), (29), (33) convergem se e somente se $p > 1$; (35) converge para qualquer valor de $p > 0$; (42) converge se e só se $0 \leq a < 1$; (37) diverge para qualquer $p > 0$; as demais são todas convergentes.